Statistical Mechanics and Quantum Integrability Systems Seminar Outline

Fall 2023

- The book doesn't explain all details. In your presentation, your job is to FILL IN those details.
- If you have any math related questions please email me ccl2166@columbia.edu IMMEDIATELY as I will need time to prepare an answer.
- Feel free to skip any physics (assuming you can distinguish between physics and math).
- The page numbering below is by what's written in the book, not the pdf which can be found in Files in Courseworks. For example, page 2 of the pdf corresponds to page 4 of the book.

1. $1D-$ Solved Models

(1) **The Ising Model**

(a) Define $\cosh(x), \sinh(x), \tanh(x)$ and show that $\cosh^2(x) - \sinh^2(x){=}1$ and thus the name "hyperbolic sine", etc.

(b) Follow [11, Chapter 2.1] up to page 50.

(c) Follow [11, Chapter 2.2] up to the end of 2.2.2. Leave the part about the Quantum Hamiltonian to the end.

(2) **The Chromatic Polynomial and the Four-Color Theorem**

(a) Follow [2, Sections 1, 2] skipping Theorem 2.6.

(b) Follow [11, Appendix 2.C] starting on page 95 and the end of [11, Section 2.5] on page 73.

(c) Prove your favorite theorems from [2, Sections 3].

(3) **Series Expansion and the Potts Model**

(a) Follow [11, Chapter 2.3]. Chapter 2.3

(b) Follow [11, Chapter 2.5]

(5) **Bessel functions and $O(n)$ symmetry**

(a) Follow [11, Appendix 2.A] up to the end of page 87 where it says, "For that reason, the $\Gamma(z)$ function ... of the factorial skipping the paragraph "Since $\Gamma(z+1)$ is defined..." Plotting Fig. 2.18 on page 89 would also be nice.

(b) Follow [11, Appendix 2.A, The Bessel functions $I_v(x)$] starting on page 90 and ending on page 92.

(c) Follow [11, Section 2.6] up to the end of page 76.

(d) Check that $I_v(x)$ is indeed a solution the "Bessel equation" (2.A.10). Proofs for the recurrences can be found in [8].

(14) **Z_n symmetry and more on the Gamma Function**

(a) Follow [11, Section 2.7]

(b) Follow [9] Section 15.8 stopping at the end of 15.8.1.

2. $2D-$ Solved Models

(4) **Duality in the $2D-$ Ising Model**

(a) Follow [11, Chapter 4.2].

(b) If you have time cover [11, Chapter 4.3].

(6) **Dimer Solution and Circulant Matrices**

(a) Follow [11, Exercise 5.4] on page 210. (Ask Cailan for notes)

(b) Follow [1] Sections 1-2 ignoring any computer code.

(7) **Pfaffians and the Dimer solution of the $2D-$Ising Model**

(a) Follow [11, Chapter 5.2].

(b) Define the matrix J, state Theorem 3 and then prove Theorem 4 in [4].

(8) **The Transfer Matrix Approach to the $2D-$Ising Model**

(a) Follow [11, Chapter 6] up to the end of Chapter 6.1.2. on the top of page 219.

(9) **Eigenvalues of the Transfer Matrix**

(a) Follow [11, Chapter 6.1.3-6.2].

3. Quantum Integrable Systems

(10) **Tensor Products and the Heisenberg Spin Chain**

(a) Follow [10] pages 5-8.

(b) Follow [7] Sections 1-4 skipping Section 3.1.

(c) Follow [3, Section 5.3] ignoring the part about measurements. (If you want to do this part, look up "Partial measurements" by Michael Nielsen on Youtube and then do Example 4 in [3].

(11) **The XXX spin chain and Coordinate Bethe Ansatz**

(a) Follow [7] Section 5.

(12) **The Lax and Monodromy Matrices and the Yang-Baxter Equation**

(a) Follow [6] up to the top of page 8, first pargraph where it says, "which he calls the star-triangle relation."

(13) **Algebraic Bethe Ansatz and the XXX spin chain**

(a) Follow [6] Sections 6-7.

(15) **The 6 Vertex Model**

(a) Follow [5] Section 6 up to Equation (6.19). Copy equation (6.21) and equation (6.25) and then start again at Section 6.3.4.

References

[1] ??? Circulant Matrices. https://web.mit.edu/18.06/www/Spring17/Circulant-Matrices.pdf.

[2] A. Aydelotte. An Exploration of the Chromatic Polynomial. https://scholarworks.boisestate.edu/cgi/viewcontent.cgi?article=1006&context=math_undergraduate_theses.

[3] J. F. Biasse. Tensor Product and Mutli-Qubit Systems. https://www.usf-crypto.org/wp-content/uploads/2021/01/Tensor_product.pdf.

[4] H. E. Haber. Notes on antisymmetric matrices and the pfaffian. https://scipp.ucsc.edu/~haber/webpage/pfaffian2.pdf.

[5] J. L. Jacobsen and Y. Ikhlef. Alg‘ebres, Int´egrabilit´e et Mod‘eles Exactement Solubles. https://www.phys.ens.fr/~jacobsen/AIMES/AIMES-complete-lecture-notes.pdf.

[6] Y. Jiang. Algebraic Bethe ansatz. https://yauc.seu.edu.cn/_upload/article/files/0b/e0/206fa9724645829f4e9b4c29aed9/3b5012a6-976f-41fe-8928-1a22c59caf89.pdf.

[7] Y. Jiang. Heisenberg spin chain and Bethe ansatz. https://yauc.seu.edu.cn/_upload/article/files/0b/e0/206fa9724645829f4e9b4c29aed9/1dacdffb-9dcb-4fde-8646-6e66794c8d76.pdf.

[8] J. Lambers. Bessel Functions of the First Kind. https://www.math.usm.edu/lambers/mat415/notes415_1004.pdf.

[9] S. Miller. The Gamma Function and Related Distributions. https://web.williams.edu/Mathematics/sjmiller/public_html/372Fa15/handouts/GammaFnChapter_Miller.pdf.

[10] H. Murayama. Notes on Tensor product. http://hitoshi.berkeley.edu/221A/tensorproduct.pdf.

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